Newton's Academy
MATHEMATICS AND STATISTICS

Time: 3 Hrs.   **Max. Marks: 80**

General instructions:
*The question paper is divided into **FOUR** sections.*

(1) **Section A:** Q.1 contains **Eight** multiple choice type of questions, each carrying **Two** marks.
                     Q.2 contains **Four** very short answer type questions, each carrying **one** mark.
(2) **Section B:** Q.3 to Q. 14 contain **Twelve** short answer type questions, each carrying **Two** marks. (Attempt any **Eight**)
(3) **Section C:** Q.15 to Q. 26 contain **Twelve** short answer type questions, each carrying **Three** marks. (Attempt any **Eight**)
(4) **Section D:** Q.27 to Q. 34 contain **Eight** long answer type questions, each carrying **Four** marks. (Attempt any **Five**)
(5) Use of log table is allowed. Use of calculator is not allowed.
(6) Figures to the right indicate full marks.
(7) Use of graph paper is <u>not</u> necessary. Only rough sketch of graph is expected.
(8) For each multiple choice type of question, it is mandatory to write the correct answer along with its alphabet, e.g. (a)……. / (b)…….. /(c)…….. /(d)……., etc. No marks shall be given, if <u>ONLY</u> the correct answer or the alphabet of correct answer is written. Only the first attempt will be considered for evaluation.
(9) Start answer to each section on a new page.

SECTION – A

Q.1. Select and write the correct answer for the following multiple choice type of questions: **[16]**

(i) The negation of $p \wedge (q \to r)$ is _____.
  (a) $\sim p \wedge (\sim q \to \sim r)$ (b) $p \vee (\sim q \vee r)$
  (c) $\sim p \wedge (\sim q \to r)$ (d) $p \to (q \wedge \sim r)$ (2)

(ii) In ΔABC if $c^2 + a^2 - b^2 = ac$, then $\angle B =$ _____.
  (a) $\dfrac{\pi}{4}$ (b) $\dfrac{\pi}{3}$ (c) $\dfrac{\pi}{2}$ (d) $\dfrac{\pi}{6}$ (2)

(iii) Equation of line passing through the points $(0, 0, 0)$ and $(2, 1, -3)$ is _____.
  (a) $\dfrac{x}{2} = \dfrac{y}{1} = \dfrac{z}{-3}$ (b) $\dfrac{x}{2} = \dfrac{y}{-1} = \dfrac{z}{-3}$
  (c) $\dfrac{x}{1} = \dfrac{y}{2} = \dfrac{z}{3}$ (d) $\dfrac{x}{3} = \dfrac{y}{1} = \dfrac{z}{2}$ (2)

(iv) The value of $\hat{i} \cdot \left(\hat{j} \times \hat{k}\right) + \hat{j} \cdot \left(\hat{k} \times \hat{i}\right) + \hat{k} \cdot \left(\hat{i} \times \hat{j}\right)$ is _____.
  (a) 0   (b) -1 (c) 1   (d) 3   (2)

(v) If $f(x) = x^5 + 2x - 3$, then $(f^{-1})'(-3) =$ _____.
  (a) 0   (b) -3 (c) $-\dfrac{1}{3}$ (d) $\dfrac{1}{2}$ (2)

(vi) The maximum value of the function $f(x) = \dfrac{\log x}{x}$ is _____.
  (a) e (b) $\dfrac{1}{e}$ (c) e^2 (d) $\dfrac{1}{e^2}$ (2)

(vii) If $\int \dfrac{dx}{4x^2 - 1} = A \log\left(\dfrac{2x-1}{2x+1}\right) + c$, then $A =$ _____.
  (a) 1   (b) $\dfrac{1}{2}$ (c) $\dfrac{1}{3}$ (d) $\dfrac{1}{4}$ (2)

(viii) If the p.m.f of a r.v.X is

$P(x) = \dfrac{c}{x^3}$, for $x = 1, 2, 3$

$= 0$, otherwise,

then $E(X) = $ _____

(a) $\dfrac{216}{251}$ (b) $\dfrac{294}{251}$ (c) $\dfrac{297}{294}$ (d) $\dfrac{294}{297}$ (2)

Q.2. Answer the following questions: **[4]**

(i) Find the principal value of $\cot^{-1}\left(\dfrac{-1}{\sqrt{3}}\right)$ (1)

(ii) Write the separate equations of lines represented by the equation $5x^2 - 9y^2 = 0$ (1)

(iii) If $f'(x) = x^{-1}$, then find $f(x)$ (1)

(iv) Write the degree of the differential equation

$\left(y'''\right)^2 + 3\left(y''\right) + 3xy' + 5y = 0$ (1)

SECTION – B

Attempt any EIGHT of the following questions: **[16]**

Q.3. Using truth table verify that:

$(p \wedge q) \vee \sim q \equiv p \vee \sim q$ (2)

Q.4. Find the cofactors of the elements of the matrix $\begin{bmatrix} -1 & 2 \\ -3 & 4 \end{bmatrix}$ (2)

Q.5. Find the principal solutions of $\cot \theta = 0$ (2)

Q.6. Find the value of k, if $2x + y = 0$ is one of the lines represented by $3x^2 + kxy + 2y^2 = 0$ (2)

Q.7. Find the cartesian equation of the plane passing through A(1, 2, 3) and the direction ratios of whose normal are 3, 2, 5. (2)

Q.8. Find the cartesian co-ordinates of the point whose polar co-ordinates are $\left(\dfrac{1}{2}, \dfrac{\pi}{3}\right)$. (2)

Q.9. Find the equation of tangent to the curve $y = 2x^3 - x^2 + 2$ at $\left(\dfrac{1}{2}, 2\right)$. (2)

Q.10. Evaluate: $\displaystyle\int_0^{\frac{\pi}{4}} \sec^4 x \ dx$ (2)

Q.11. Solve the differential equation $y\dfrac{dy}{dx} + x = 0$ (2)

Q.12. Show that function $f(x) = \tan x$ is increasing in $\left(0, \dfrac{\pi}{2}\right)$. (2)

Q.13. From the differential equation of all lines which makes intercept 3 on x-axis. (2)

Q.14. If $X \sim B(n, p)$ and $E(X) = 6$ and $Var(X) = 4.2$, then find n and p. (2)

SECTION – C

Attempt any EIGHT of the following questions: **[24]**

Q.15. If $2\tan^{-1}(\cos x) = \tan^{-1}(2\cosec x)$, then find the value of x. (3)

Q.16. If angle between the lines represented by $ax^2 + 2hxy + by^2 = 0$ is equal to the angle between the lines represented by $2x^2 - 5xy + 3y^2 = 0$, then show that $100\left(h^2 - ab\right) = \left(a + b\right)^2$. (3)

Q.17. Find the distance between the parallel lines $\dfrac{x}{2} = \dfrac{y}{-1} = \dfrac{z}{2}$ and $\dfrac{x-1}{2} = \dfrac{y-1}{-1} = \dfrac{z-1}{2}$. (3)

Q.18. If A (5, 1, p), B(1, q, p) and C(1, –2, 3) are vertices of a triangle and $G\left(r, \dfrac{-4}{3}, \dfrac{1}{3}\right)$ is its centroid, then find the values of p, q, r by vector method. (3)

Q.19. If $A(\overline{a})$ and $B(\overline{b})$ be any two points in the space and $R(\overline{r})$ be a point on the line segment AB dividing it internally in the ratio m : n then prove that $\overline{r} = \dfrac{m\overline{b} + n\overline{a}}{m+n}$. (3)

Q.20. Find the vector equation of the plane passing through the point A(–1, 2, –5) and parallel to the vectors $4\hat{i} - \hat{j} + 3\hat{k}$ and $\hat{i} + \hat{j} - \hat{k}$. (3)

Q.21. If $y = e^{m \tan^{-1}x}$, then show that $(1+x^2)\dfrac{d^2 y}{dx^2} + (2x-m)\dfrac{dy}{dx} = 0$ (3)

Q.22. Evaluate: $\int \dfrac{dx}{2 + \cos x - \sin x}$ (3)

Q.23. Solve $x + y\dfrac{dy}{dx} = \sec(x^2 + y^2)$ (3)

Q.24. A wire of length 36 meters is bent to form a rectangle. Find its dimensions if the area of the rectangle is maximum. (3)

Q.25. Two dice are thrown simultaneously. If X denotes the number of sixes, find the expectation of X. (3)

Q.26. If a fair coin is tossed 10 times. Find the probability of getting at most six heads. (3)

<div align="center">SECTION – D</div>

Attempt any FIVE of the following questions: **[20]**

Q.27. Without using truth table prove that
$(p \wedge q) \vee (\sim p \wedge q) \vee (p \wedge \sim q) \equiv p \vee q$ (4)

Q.28. Solve the following system of equations by the method of inversion
$x - y + z = 4,\ 2x + y - 3z = 0,\ x + y + z = 2$ (4)

Q.29. Using vectors prove that the altitudes of a triangle are concurrent. (4)

Q.30. Solve the L.P.P. by graphical method,
Minimize $\quad z = 8x + 10y$
Subject to $\quad 2x + y \geq 7,$
$\qquad\qquad 2x + 3y \geq 15,$
$\qquad\qquad y \geq 2, x \geq 0$ (4)

Q.31. If $x = f(t)$ and $y = g(t)$ are differentiable functions of t so that y is differentiable function of x and $\dfrac{dx}{dt} \neq 0,$ then prove that:

$$\dfrac{dy}{dx} = \dfrac{\dfrac{dy}{dt}}{\dfrac{dx}{dt}}$$

Hence find $\dfrac{dy}{dx}$ if $x = \sin t$ and $y = \cos t$. (4)

Q.32. If u and v are differentiable function of x, then prove that:

$$\int uv\, dx = u\int v\, dx - \int\left[\dfrac{du}{dx} \cdot \int v\, dx\right] dx$$

Hence evaluate $\int \log x\, dx$ (4)

Q.33. Find the area of region between parabolas $y^2 = 4ax$ and $x^2 = 4ay$. (4)

Q.34. Show that: $\displaystyle\int_{0}^{\frac{\pi}{4}} \log(1 + \tan x)\, dx = \dfrac{\pi}{8}\log 2$ (4)